Combinatorics and Algebras from A to Z

Titles and Abstracts

Eli Aljadeff

PI Theory and G-graded Division Algebras

If G is a finite group and D is a finite dimensional G-graded division algebra over k, then restricting scalars to $F = k$, the algebraic closure, we obtain a finite dimensional G-graded simple algebra.

In this lecture we address the problem in the opposite direction, namely if A is a finite dimensional G-graded simple algebra over F (with $char(F) = 0$), then under which conditions does it admit a G-graded division algebra form (in the sense of descent theory)?

The main tools come from G-graded PI theory. These allow us to construct the corresponding generic objects.

Joint work with Karasik.

Allan Berele

Specialized Symmetric Polynomials

The problem of determining the relations between symmetric functions in a finite number of not necessarily distinct variables is related to the problem of determining trace identities for diagonal matrices, with a non-standard trace function.

The talk will be elementary, with a number of open problems.

Shaoshi Chen

A Residue-Based Approach to Vanishing Sums

Residue theorems have been used extensively to evaluate definite integrals and sums in combinatorics and special functions.

In this talk, we will first give an overview of several methods for proving combinatorial identities, including the celebrated Wilf-Zeilberger method, and then present a new approach to proving and discovering vanishing sums based on a residue theorem due to Nicole in 1717.

This is a joint work with Rong-Hua Wang.

Miriam Cohen

Conjugacy Classes and Representations of Hopf Algebras and their Quantum Double

We assume throughout that Hopf algebras H are semisimple algebras (hence finite dimensional) over a field of characteristic 0. Conjugacy classes and conjugacy sums for Hopf algebras were defined in our previous work. They boil down to the usual ones when applied to finite group algebras. An essential property of the set of conjugacy classes $\{\mathcal{C}_i\}$ is that they are irreducible representations of the quantum double of H, $D(H)$. Moreover, when H is quasitriangular, then $H = \bigoplus_{i=1}^{n} \mathcal{C}_i$ as $D(H)$-modules.

We focus on quasitriangular Hopf algebras and show that irreducible representations of $D(H)$ are all obtained from tensor product of irreducible H-modules and conjugacy classes. That is:

Theorem: Let H be a semisimple quasitriangular Hopf algebra over a field of characteristic 0. Then irreducible representations of $D(H)$ are obtained as direct summands of $V_i \otimes \mathcal{C}_j$ where V_i is an irreducible representation of H and $\mathcal{C}_j$ is a conjugacy class of H.

When V_i is 1-dimensional then $V_i \otimes \mathcal{C}_j$ is a certain deformation of $\mathcal{C}_j$ which is also an irreducible representation of $D(H)$.

Joint work with Sara Westreich.

Aviezri Fraenkel

Patrolling the Border of a Striking Conjecture

We discuss the following conjecture concerning disjointness and covering of integers: There is a unique system of sequences involving powers of 2 with distinct moduli, covering all integers.

One of our main results is that the conjecture almost holds if powers of 2 are replaced by any power of a number greater than 2. In that case, two of the sequences necessarily intersect, but the others can be disjoint.

When two sequences intersect, some numbers must be missing from the union of all sequences, since the reciprocals of all the moduli sum to 1 by density considerations. We study the structure of the complementary sequence.

Maria Gorelik

Superalgebras Described by Root Data

Similarly to the classical theory of semisimple (or Kac-Moody) Lie algebras, we describe a procedure to assign a Lie superalgebra to certain root data. Our notion of root data takes into account the existence of "odd reflections", which are important in the theory of Lie superalgebras. This approach produces so-called "root algebras" such that Kac-Moody (super)algebras are the "minimal ones".

The talk is based on a joint project with V. Hinich and V. Serganova.

Qing-Hu Hou

Log-convexity of P-recursive sequences

Based on the asymptotic analysis, we present a method for proving the log-convexity or the log-concavity of P-recursive sequences.

Gil Kalai

My Wonderful Experience with Enumeration, Computers, Algebra, Z, and A

I will talk about personal (amateurish at times) experience on how adding weights can help us enumerate, how computers can explore and prove theorems, and how algebra and combinatorics interlace.

Manuel Kauers

Quadrant Walks Starting Outside the Quadrant

Throughout his long academic life, Doron has always been very good at thinking ``outside the box''. I am going to present a study of a combinatorial problem where we literally also were thinking ``outside the box'': we consider lattice walks restricted to a quadrant whose starting point is outside the quadrant. This model leads to an interesting functional equation that at first glance looks like it should have an algebraic solution. With the help of the celebrated creative telescoping technology pioneered by Doron, we could show that the solution is in fact transcendental. For certain variations of the problem, we have semi-rigorous proofs that the generating functions are also transcendental, and in one case, we suspect that the generating function is not even D-finite.

This is joint work with Manfred Buchacher and Amelie Trotignon, to be published at FPSAC 2021.

Christoph Koutschan

Binomial Determinants for Tiling Problems Yield to the Holonomic Ansatz

We study some families of binomial determinants with signed Kronecker deltas that are located along an arbitrary diagonal in the corresponding matrix. They count cyclically symmetric rhombus tilings of hexagonal regions with triangular holes. By adapting Zeilberger's holonomic ansatz to make it work for these problems, we can take full advantage of computer algebra tools for symbolic summation. As a result, we are able to resolve all open conjectures related to these determinants, including one from 2005 due to Lascoux and Krattenthaler.

Toufik Mansour

Restricted Permutations, Conjecture of Lin and Kim, and Work of Andrews and Chern

In each of the following cases, we will see how computer programming assists with finding the solution.

1. Let w_k be the number of distinct Wilf classes of subsets of exactly k patterns in $\mathfrak{S}_4$. We show that $w_3 = 242$, $w_4 = 1100$, $w_5 = 3441$, $w_6 = 8438$, $w_7 = 15392$, $w_8 = 19002$, $w_9 = 16293$, $w_{10} = 10624$, $w_{11} = 5857$, $w_{12} = 3044$, $w_{13} = 1546$, $w_{14} = 786$, $w_{15} = 393$, $w_{16} = 198$, $w_{17} = 105$, $w_{18} = 55$, $w_{19} = 28$, $w_{20} = 14$, $w_{21} = 8$, $w_{22} = 4$, $w_{23} = 2$, and $w_{24} = 1$.
2. We compute the distribution of the first letter statistic on nine avoidance classes of permutations, corresponding to two pairs of patterns of length 4. In particular, we show that the distribution is the same for all classes, and is given by a new Schrőder number triangle. This answers in the affirmative a recent conjecture of Lin and Kim.
3. In a recent paper by Andrews and Chern, it was shown that the distribution of asc, the number of ascents on the inversion sequence avoidance class $I_n(\geq, \neq, >)$, is the same as that of $n - 1 - \mathrm{asc}$ on the class $I_n(>, \neq, \geq)$. By employing recurrence relations and then a functional equations approach together with the kernel method, we are able to compute explicitly the generating function for both of the aforementioned joint distributions, which extends the recent result $|I_n(\geq, \neq, >)| = |I_n(>, \neq, \geq)|$.

Aaron Robertson

The Thread of Ramsey Theory in Zeilberger's Work

Zeilberger gave me a Ramsey theory problem as part of my dissertation. I always wondered why, since he did not do Ramsey theory, and all of his other students at around that time were working in the area of permutation patterns.

Follow me as I attempt to discover from where this may have evolved.

Thotsaporn "Aek" Thantipanonda

The Card Guessing Game: A Generating Function Approach

Consider a card guessing game with complete feedback, in which a deck of n cards, labeled $1, \dots, n,$ is riffle-shuffled once. With the goal to maximize the number of correct guesses, a player guesses the cards from the top of the deck, one at a time, under the optimal strategy until no cards remain.

We provide the statistics (e.g. mean, variance, etc.) of the number of correct guesses.

Vince Vatter

Growth Rates of Grids and Merges of Permutation Classes

The first general result on growth rates of permutation classes is a consequence of Regev's foundational 1981 work on the asymptotic enumeration of Young diagrams, and gives the growth rate of the class of permutations avoiding any monotone pattern.

I will describe a generalization of a theorem of Bevan, which shows that the growth rate of a grid class of permutations is given by the square of the largest singular value of a related matrix. I will then show how this result can be used to rederive several consequences of Regev's results, as well as establish several other results of Bóna and others.

Joint work with Michael Albert (University of Otago) and Jay Pantone (Marquette University)